**Enrollment No: _______________________ Exam Seat No: _______________________**

# C.U.SHAH UNIVERSITY

## Summer Examination-2018

**Subject Name: Graph Theory**

**Subject Code:4SC06GTC1** **Branch: B.Sc. (Mathematics)**

**Semester: 6** **Date:04/05/2018** **Time:02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

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**Q-1** **Attempt the following questions:** **(14)**

**a)** Define: Pendant vertex. (01)

**b)** If $G$ is a simple graph with n vertices, then find the maximum number of edges. (01)

**c)** Define: Spanning subgraph. (01)

**d)** Every closed walk is cycle. Determine whether the statement is True or False. (01)

**e)** A vertex with minimum eccentricity is called___________. (01)

**f)** If $G$ is a__________ graph if and only if it has exactly two vertices of odd degree. (01)

$(i)$ disconnected $(iii)$ unicursal
$(ii)$ Euler $(iv)$ none of these

**g)** Define: Open walk. (01)

**h)** For which values of $m$ and $n$, $K_{m,n}$ is Euler graph? (01)

**i)** $W_5$ is a Hamiltonian graph. Determine whether the statement is True or False. (01)

**j)** The rank of connected graph is ___________. (01)

**k)** If $G$ be an acyclic graph with $n$ vertices and $k$-components, then $G$ has ______edges. (01)

**l)** How many cut vertices exist in complete graph? (01)

**m)** Draw the 4-regular simple graph with 6 vertices. (02)

**Attempt any four questions from Q-2 to Q-8**

**Q-2** **Attempt all questions** **(14)**

**a)** State and prove first theorem of graph theory. Using it prove that the number of odd vertices in graph is even. (07)

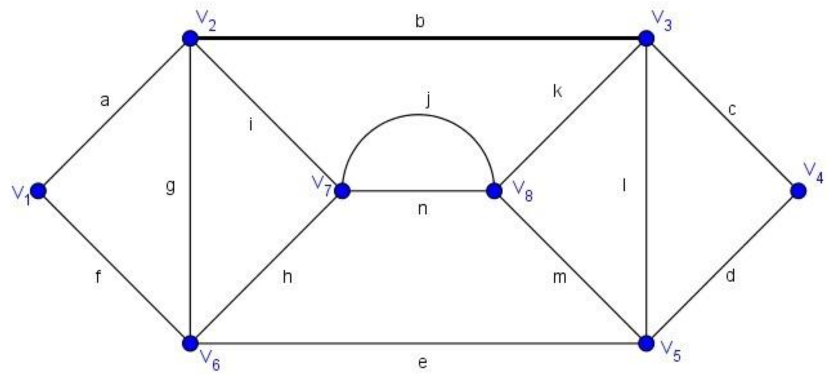
**b)** Let $G = (V, E)$ be a $k$-regular graph where $k$ is an odd number.Then prove that number of edges in graph $G$ is in multiple of $k$. (05)

**c)** Draw a graph with degree sequence 0, 1, 2, 3, 3, 4, 5, 6. (02)

**Q-3** **Attempt all questions** **(14)**

**a)** State and prove necessary and sufficient condition for disconnected graph. (07)

**b)** Answer the following questions from the given graph. (07)




$(i)$ Write one cycle from $V_7$ with length 8.

$(ii)$ Find edges in series.

$(iii)$ Write one path of length 7.

$(iv)$ How many odd and even vertices in the graph?

$(v)$ Write one closed walk with length 9 which is not cycle.

$(vi)$ Write one Euler line.

**Q-4** **Attempt all questions** **(14)**

**a)** Let $G$ be a simple graph with $n$ vertices and $k$-components.Thenprove that $G$ have at most $\frac{(n-k)(n-k+1)}{2}$ number of edges. (07)

**b)** Draw the dodecahedron graph and find Hamiltonian cycle in it, if exist. (05)

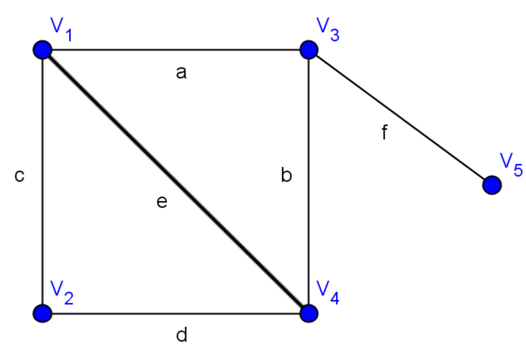
**c)** What is the smallest positive integer $n$ such that the complete graph has at least 1000 edges? (02)

**Q-5** **Attempt all questions** **(14)**

**a)** State and prove Euler's theorem. (07)

**b)** Prove that$K_{2,3} \cong K_{3,2}$. (05)

**c)** Find a fusion graph of the following graph by fusing vertex $V_1$ and $V_2$. (02)



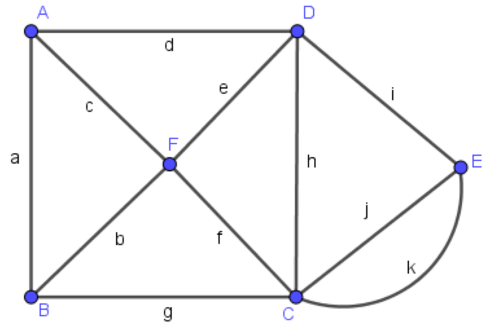

**Q-6** **Attempt all questions** **(14)**

**a)** If $G$be a tree with n vertices, then prove that it has $n-1$ edges. (05)

**b)** Without drawing graph check whether the graph corresponding to the following adjacency matrix is connected or not: (05)

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

**c)** Write four fundamental cut sets and four fundamental circuits with respect to spanning tree $T = \{a, d, f, i, k\}$of the following graph. (04)

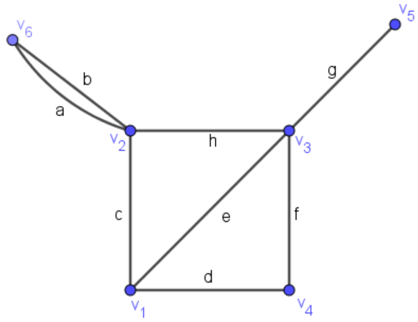



**Q-7** **Attempt all questions** **(14)**

**a)** Prove that in a complete graph with $n$ vertices, there are $\left(\frac{n-1}{2}\right)$ edge-disjoint Hamiltonian circuits, if $n$ is an odd number greater than equal to 3. (06)

**b)** Prove that graph G is a tree if and only if it is minimal connected graph. (04)

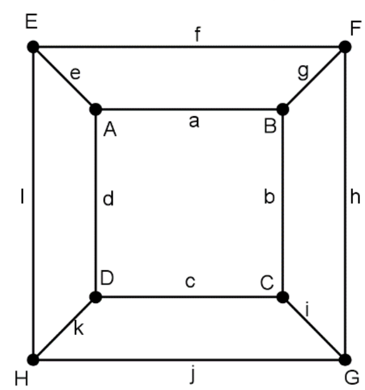
**c)** Find path matrix for $P(v_1, v_5)$and circuit matrix for following graph (04)




**Q-8** **Attempt all questions** **(14)**

**a)** Explain Konigsberg bridge problem and write solution which is given by Euler. (06)

**b)** Define incidence matrix and find it for following graph. (04)




**c)** Find the number of pendant vertices in binary tree with $n$ vertices. (04)